# Nonadiabatic decoherence control of qubits strongly coupled to continuum edge

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We propose a method for contolling the decoherence of a driven qubit that is strongly coupled to a reservoir, when the qubit resonance frequency is close to a continuum edge of the reservoir spectrum. This strong-coupling regime is outside the scope of existing methods of decoherence control. We demonstrate that an appropriate sequence of nearly abrupt changes of the resonance frequency can protect the qubit state from decay and decoherence more effectively than the intuitively obvious alternative, which is to fix the resonance well within a forbidden bandgap of the reservoir spectrum, as far as possible from the continuum edge. The "counterintuitive" nonadiabatic method outlined here can outperform its adiabatic counterparts in maintaining a high fidelity of quantum logic operations. The remarkable effectiveness of the proposed method, which requires much lower rates of frequency changes than previously proposed control methods, is due to the ability of appropriately alternating detunings from the continuum edge to augment the interference of the emitted and back-scattered quanta, thereby helping to stabilize the qubit state against decay. Applications to the control of decoherence near the edge of radiative, vibrational and photoionization continua are discussed.

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## I. INTRODUCTION

The interaction between a spectrally structured field-continuum and a two-level system (TLS), which can be "dressed by coupling to near-resonant field modes, defines a class of fundamentally important processes in quantum field theories, encompassing quantum electrodynamics [1] and collective excitations in condensed media [2]. A diversity of peculiar features in the TLS and field dynamics has been shown [3, 4, 5, 6, 7, 8] to arise when a bare or dressed TLS resonance is close to the continuum edge of the field spectrum. These features stem from the strong coupling between the TLS and a continuum of field modes with an abrupt (or nearly abrupt) cut-off and the resulting oscillatory *non-Markovian* character of decay into the continuum. Perhaps the most spectacular feature is the possibility of forming a *discrete "dressed" state*, whose energy lies outside the continuum, in the forbidden bandgap, and is stable against decay [3]. The excited TLS then evolves into a superposition of a discrete, stable state with energy in the forbidden bandgap and a decaying excited-state component with energy in the continuum. The latter effect should be manifest in various systems: (i) the partial inhibition of radiative decay in photonic crystals [4, 5, 6] and high-$Q$ cavities [7, 8, 9]; (ii) the stabilization of a local vibrational mode against decay near the Debye cut-off of the phonon spectrum in a solid [10]; an ion trap [11] or an optical lattice [12], or (iii) the stabilization of an electronic state against auto-ionization [13]. It can be interpreted as an "anomalous Lamb shift" [4, 5, 7, 8] of the discrete, excited, level by the emitted quantum that pushes the level beyond cut-off, or as interference of the emitted quantum with its backscattered amplitude that localizes (binds) the excitation to the TLS.

The present paper is dedicated to the challenge of coherent, dynamical, control of decay and decoherence in a TLS (a qubit) whose resonance frequency is close to a continuum edge of a reservoir, allowing for the possibility of strong coupling between the TLS and the reservoir. We raise the question: how can decoherence control benefit from the discrete, stable state of the excited TLS in the forbidden bandgap? Spontaneous emission (SE) of quanta (*e.g.*, phonons or photons) into the continuum of the reservoir is the dominant source of decoherence at low temperatures, as the occupancy of the reservoir modes becomes weak [15, 16]. A promising means of protection from SE is to have the TLS resonance frequency well within the bandgap: for example, to embed the atoms in photonic crystals (three-dimensionally periodic dielectric structures) so as to block atomic SE at frequencies within the spectrally-wide, omnidirectional photonic bandgaps (PBGs) [17, 18, 19]. Highly localized electronic or vibronic states decoupled from their continua would be similarly immune to SE of their respective quanta. Yet qubit manipulations for quantum information processing may necessitate TLS transition frequencies near the edge of the continuum, where SE is only *partially* blocked [4, 5]. Thus, in order to coherently manipulate an atomic transition in the PBG via a single-photon (rather than two-photon) transition, one must take advantage of its proximity to the edge frequency and couple it to a field mode in the continuum or to a mode in the PBG created by a local defect in the photonic crystal [5, 20] (Fig. 1-Inset). Similar considerations apply to qubits that must be intermittently coupled to vibrational continua, *e.g.*, trapped ions [11], or atoms in optical lattices [23].

In order to operate quantum logic gates, based on pairwise *entanglement* of qubits, say by resonant dipole-dipole interactions [22], or by resonant exchange of a phonon in an ion-trap [11] or of an electron between quantum dots [12] one should be able to switch the in-

teraction on- and off-, *e.g.*, by AC Stark-shifts of the transition frequency of one qubit relative to the other. In structured continua, this would significantly change the detuning of the resonance frequency from the continuum edge, and thereby the degree of SE blocking. The question then arises: should such frequency shifts be performed adiabatically? The answer is expected to be affirmative, based on existing treatments of adiabatic entanglement and protection from decoherence [24, 25] and on the tendency of nonadiabatic evolution to *spoil the gate fidelity* and promote transitions to the continuum [26]. Surprisingly, the present analysis demonstrates that an appropriate sequence of *"sudden"* (strongly nonadiabatic) changes of the detuning from the continuum edge may yield *higher fidelity* of qubit and quantum gate operations than their adiabatic counterparts. This unconventional nonadiabatic protection from decoherence can be attributed to the ability of appropriately alternating detunings from the continuum edge in the strong-coupling regime to *augment the interference* of the emitted and back-scattered quanta amplitudes, thereby increasing the probability amplitude of the stable state.

The present method is very different from previous proposals [27, 28, 29] to suppress decoherence by ultra fast measurements or modulation of the coupling with the continuum, which are all included in the *universal formula* of Ref. [28]. In order to effect decoherence suppression akin to the quantum Zeno effect (QZE) [16, 28], the measurements or modulation must be repeated at a rate exceeding the spectra width of the continuum (its inverse spectral width) which may be prohibitive: $10^{18}$ s$^{-1}$ for radiative continua [16], and $10^{13}$ s$^{-1}$ for vibrational continua [10]. By contrast, the present method requires frequency shifts at a rate comparable to the energy of the qubit-continuum interaction at the edge, which is typically *much lower* than the inverse memory time of the continuum. Furthermore, the ultrafast modulation (QZE) strategy is valid only when the coupling to the continuum is weak, *i.e.*, far enough from the continuum edge, whereas the present method works equally well for strong or weak coupling to the continuum, since it is based on *phase-dependent* changes of the qubit state that is "dressed" by the continuum, rather than on modulation of the "bare" state that is weakly perturbed by the "bath" [27, 28, 29].

In Sec. II we revisit the equations of motion for partial decay of states near the continuum edge, allowing for the possibility of strong coupling. In Sec. III we introduce appropriate dynamical sequences of sudden frequency changes for the control of such decay. In Sec. IV we discuss their implications for quantum gates and in Sec. V the approximations involved in our quantitative analysis. Experimental realizations and conclusions are discussed in Sec. VI.

## II. HAMILTONIAN AND EQUATIONS OF MOTION

We consider a TLS with excited and ground states $|e\rangle$ and $|g\rangle$ with linear (dipolar) coupling to the field of a discrete (or defect) mode and to a spectrally-structured mode continuum (*e.g.*, radiative continuum in a photonic crystal [17] or a high-$Q$ cavity [7, 8, 9], a structured vibrational continuum in an ion trap or optical lattice [10, 11], or an electronic continuum in a nanostructure [23]). The hamiltonian of the system in the rotating-wave approximation assumes the form [5]

$$\begin{aligned} H = \hbar\omega_{at}\ |e\rangle\langle e| + \hbar \int_0^{+\infty} \omega\ a_\omega^\dagger a_\omega\ \rho(\omega)\ d\omega \\ + \hbar\left(\kappa_d^\star\ a_d^\dagger\ |g\rangle\langle e| + h.c.\right) \\ + \hbar \int_0^{+\infty} \left[\kappa^\star(\omega)\ a_\omega^\dagger\ |g\rangle\langle e| + h.c.\right] \rho(\omega)\ d\omega. \end{aligned} \tag{1}$$

Here $\hbar\omega_{at}$ is the energy of the atomic transition frequency, $a_\omega^\dagger$ and $a_\omega$ are, respectively, the creation and annihilation operators of the field mode at frequency $\omega$, $\rho(\omega)$ is the mode density of the continuum, the coupling rates between the dipole and a mode from the continuum or a discrete mode are $\kappa(\omega)$ and $\kappa_d$, respectively.

Let us first consider the initial state obtained by absorbing a quantum from the discrete mode:

$$|\Psi(0)\rangle = |e, \{0_\omega\}\rangle, \tag{2}$$

where $|\{0_\omega\}\rangle$ is the vacuum state of the field. Then the evolution of the wavefunction $|\Psi(t)\rangle$ has the form

$$\begin{aligned} |\Psi(t)\rangle = \ \alpha(t)\ |e, \{0_\omega\}\rangle + \beta_d(t)\ |g,\ 1_d\rangle \\ + \int_0^{+\infty} \beta_\omega(t)\ |g, 1_\omega\rangle\ \rho(\omega)\ d\omega \end{aligned} \tag{3}$$

where we have denoted by $|1_\omega\rangle$ and $|1_d\rangle$ the single-quantum state of the relevant modes. The Schrödinger equation

$$i\dot{\Psi}(t) = H\ \Psi(t) \tag{4}$$

then leads to the set of coupled differential equations

$$\begin{aligned} \dot{\alpha}(t) = \ -i\ \omega_{at}\ \alpha(t) - i\ \kappa_d\ \beta_d(t) \\ -i \int_0^{+\infty} \kappa(\omega)\ \beta_\omega(t)\ \rho(\omega)\ d\omega, \\ \dot{\beta}_d(t) = \ -i\ \omega_d\ \beta_d(t) - i\ \kappa_d^\star\ \alpha(t), \\ \dot{\beta}_\omega(t) = \ -i\ \omega\ \beta_\omega(t) - i\ \kappa^\star(\omega)\ \alpha(t). \end{aligned} \tag{5}$$

This evolution reflects the interplay between the off-resonant Rabi oscillations of $|e,\ \{0_\omega\}\rangle$ and $|g,\ 1_d\rangle$, at the driving rate $\kappa_d$, and the *partly-inhibited oscillatory decay* from $|e,\ \{0_\omega\}\rangle$ to $|g,\ \{1_\omega\}\rangle$ via coupling to the continuum $\rho(\omega)$. This decay depends on the detuning of $\omega_{at}$ from the continuum edge at $\omega_U$ (the upper cut-off of the continuum, see Fig. 1 - inset). For a *spectrally steep* edge (see below), we are in the regime of *strong coupling* to the mode continuum (as in a high-$Q$ cavity [7, 8, 9, 19]) which allows for the existence of an oscillatory, non-decaying, component of $\alpha(t)$, associated with a discrete, stable state [5].

The possibility that a stable state exists yields the inverse Laplace transform of Eq. (5) and the corresponding wavefunction (3) in the form

$$|\Psi(t)\rangle = C^{1/2}\ |\psi_s\rangle\ \exp(-i\ \omega_0 t) + |\Psi_c(t)\rangle. \qquad (6)$$

Here the stable state has the energy $\hbar\omega_0$; $|\psi_s\rangle$ is the stable-state eigenfunction of the Hamiltonian (1) normalized to unity and weighted by the amplitude

$$C^{1/2} = \left(1 + \int_0^\infty d\omega\ \frac{|\kappa(\omega)|^2\ \rho(\omega)}{(\omega-\omega_s)^2}\right)^{-1/2}. \qquad (7)$$

The explicit form of the normalized stable eigenfunction is

$$|\psi_0\rangle = C^{1/2}\left(|e,\ \{0_\omega\}\rangle - \int_0^\infty \frac{\kappa^\star(\omega)}{\omega-\omega_0}\ |g,\ 1_\omega\rangle\ \rho(\omega)\ d\omega\right). \qquad (8)$$

It is seen to be a dressed state consisting of an excited-state component and a ground-state component, which is a superposition of contributions from all modes forming the "bound" quantum.

The other part of $|\Psi(t)\rangle$ in Eq. (5) is a superposition of the continuous-spectrum eigenfunctions of the Hamiltonian:

$$|\Psi_c(t)\rangle = \alpha_c(t)\ |e,\ \{0_\omega\}\rangle + \int_0^\infty \beta_{c\omega}(t)\ |g,\ 1_\omega\rangle\ \rho(\omega)\ d\omega, \qquad (9)$$

with a decaying excited-state amplitude

$$\alpha_c(t) = \alpha(t) - C\ \exp(-i\ \omega_0 t) \to 0 \quad \text{for } t\to\infty, \qquad (10)$$

and correspondingly increasing amplitudes of field modes in the continuum,

$$\beta_{c\omega}(t) = \beta_\omega(t) + \frac{\exp(-i\ \omega_0 t)\ C\ \kappa^\star(\omega)}{\omega-\omega_0}, \qquad (11)$$

where $\beta_\omega(t)$ is defined by Eq. (3).

Since $|\Psi(t)\rangle$ and $|\psi_0\rangle$ are all normalized to unity, it follows that the norm of the continuum-spectrum wavefunction is

$$\langle\Psi_c(t)|\Psi_c(t)\rangle = 1 - C. \qquad (12)$$

At $t\to\infty$ this norm becomes the probability of spontaneous decay, which is less than unity by virtue of the existence of stable states in the forbidden bandgaps. When there is at most one discrete stable state, this probability is non-zero, since $C<1$. This allows us to conclude that *there is always a non-vanishing* (albeit small) *probability of decay* for an excited TLS *with* $\omega_c$ *in a forbidden band gap.*

Let us now introduce abrupt changes of $\omega_{at}$, *i.e.*, of the detuning $\Delta_{at} = \omega_U - \omega_{at}$ from the upper cut-off, $\omega_U$, of the continuum (by fast AC-Stark modulations as discussed below), at intervals $\tau$. In the sudden-change approximation for $\omega_{at}$, the amplitudes $(\alpha_{dyn}(t),\ \beta_{ddyn},\ \{\beta_{\omega dyn}(t)\})$ of the excited state, the discrete mode and the continuum still evolve according to Eqs. (5), except that from $t=0$ to $t=\tau$ the atomic transition frequency is $\omega_{at}=\omega_A$, *i.e.*, the detuning $\Delta_{at} = \omega_U - \omega_A = \Delta_A$, while for $t>\tau$, we have $\omega_{at}=\omega_B$, *i.e.*, $\Delta_{at}=\Delta_B$. This dynamics leads to the relation

$$\begin{gathered}\alpha_{dyn}(t) = \alpha_A(t),\ \beta_{ddyn}(t) = \beta_{d,A}(t),\\ \beta_{\omega dyn}(t) = \beta_{\omega,A}(t),\ (t\le\tau)\ ;\\ \alpha_{dyn}(t) = \alpha_B^{(s)}(t),\ \beta_{ddyn}(t) = \beta_{d,B}^{(s)}(t),\\ \beta_{\omega dyn}(t) = \beta_{\omega,B}^{(s)}(t),\ (t>\tau).\end{gathered} \qquad (13)$$

Here both $(\alpha_A(t),\ \beta_{d,A}(t),\ \{\beta_{\omega\ A}(t)\})$ and $(\alpha_B^{(s)}(t),\ \beta_{d,B}^{(s)}(t),\ \{\beta_{\omega,B}^{(s)}(t)\})$ are solutions of Eqs. (5) with a static (fixed) atomic transition frequency, $\omega_A$ or $\omega_B$. However, the initial condition at the instant $t=\tau$ of the frequency change from $\Delta_A$ to $\Delta_B$ is no longer the excited state (2) but the superposition:

$$\begin{aligned}|\Psi(\tau)\rangle = &\ \alpha_A(\tau)\ |e,\ \{0_\omega\}\rangle + \beta_{d,A}(\tau)\ |g,\ 1_d\rangle\\ &+ \int_0^{+\infty} \beta_{\omega,A}(\tau)\ |g,\ 1_\omega\rangle\ \rho(\omega)\ d\omega.\end{aligned} \qquad (14)$$

In other words, the dynamics is equivalent to two successive static evolutions, the second one starting from initial conditions $(\alpha_A(\tau),\ \beta_{d,A}(\tau),\ \{\beta_{\omega,A}(\tau)\})$.

Using the Laplace transform of the system (5) with the initial condition (14), it is possible to express the dynamic amplitude of the excited state after the sudden change as

$$\begin{aligned}\alpha_{dyn}(t) = &\ \alpha_A(\tau)\ \alpha_B(t-\tau) + \beta_{d,A}(\tau)\ \beta_{d,B}(t-\tau)\\ &+ \int_0^{+\infty} \beta_{\omega,A}(\tau)\ \beta_{\omega,B}(t-\tau)\ \rho(\omega)\ d\omega,\ (t>\tau),\end{aligned} \qquad (15)$$

where we have used the initial conditions $(\alpha_A(\tau),\ \beta_{d,A}(\tau),\ \{\beta_{\omega,A}(\tau)\})$ and the solution $(\alpha_B(t),\ \beta_{d,B}(t),\ \{\beta_{\omega,B}(t)\})$ of Eqs. (5) for the initial condition (2).

## III. INTERFERING SUCCESSIVE EVOLUTIONS: "COUNTERINTUITIVE" SEQUENCE

Clearly, Eq. (15) is sensitive to the *relative phases* of the successive static evolutions (labelled by A and B), *i.e.*, to their *interference*. The contribution of the first term in (15) to the excited-state population $|\alpha_{dyn}(t)|^2$ always decreases after the sudden change and then oscillates, before settling to an asymptotic non-zero value. On the other hand, the contribution of the second and third terms and their cross-product with the first term increases immediately after the sudden change. Yet whatever the time $\tau$ of the sudden change, when performing only one change, the increasing contribution is never large enough to compensate for the decreasing part. Then the dynamic population of the excited state after the sudden change always lies inbetween the two static populations obtained for $\Delta_A$ and $\Delta_B$.

There is, however, an advantageous feature to the sudden change: since the time dependence of $\alpha_{dyn}(t)$ in (15) arises from the static amplitudes $\alpha_B$, $\beta_{d,B}$ and $\beta_{\omega,B}$ at the *shifted* time $t-\tau$, a consequence of the sudden change is to revive the excited-state population oscillations, which tend to disappear at long times in the static case. Hence, by applying several *successive* sudden changes, we should be able to maintain large-amplitude oscillations of the *coherence* between $|e\rangle$ and $|g\rangle$. The scenario leading to the largest amplitude consists in *periodic* shifts of the energy detuning from $\Delta_A$ to $\Delta_B$. Here we have the choice between starting from $\Delta_{at}=\Delta_A$ or $\Delta_{at}=\Delta_B$ (Fig. 1-inset). Analysis of Eq. (15) then shows that in the former case, the dynamic population experiences large amplitude oscillations but never exceeds the highest static population. But when the initial detuning $\Delta_A$ is large and we first reduce it to $\Delta_B$ before it increases to $\Delta_A$, the *dynamic population and the* $|e\rangle-|g\rangle$ *coherence*, thanks to the revival of oscillations, are *periodically larger* than the static ones (!). This is illustrated in Fig. 1 (see Sec. V for quantitative assumptions).

This remarkable result comes about unexpectedly, since it implies that successive abrupt changes can *reverse* the decay to the continuum, even though they *cannot be associated with the quantum Zeno effect* [13, 16] or with its ultrafast modulation counterparts [27, 28, 29]: the present abrupt changes occur at intervals much longer than the correlation (Zeno) time of the continuum, which is utterly negligible for the case of the radiative (electromagnetic) continuum ($10^{-18}$ s) [16]. The required intervals are even longer than the static-oscillation half-period. The fact that this happens only for the rather "counter-intuitive" ordering of detuning values (from large to small then back again) is a manifestation of interference between successive static evolutions: their relative phases determine the beating between the emitted and reabsorbed (back-scattered) photon amplitudes and thereby the oscillation of $\alpha_{dyn}(t)$.

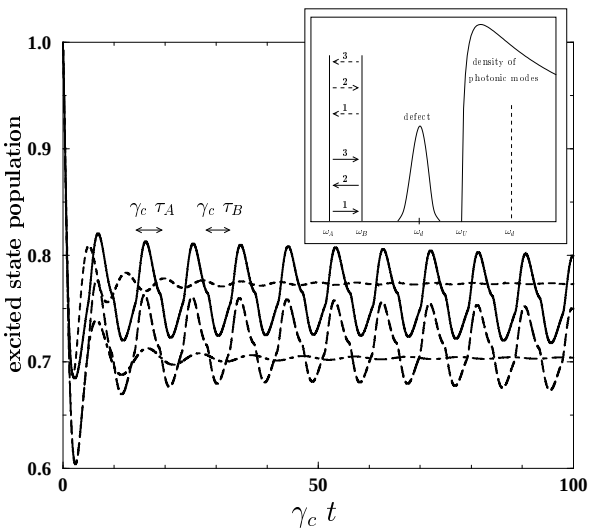


FIG. 1: Excited state population as a function of dimensionless time $\gamma_c\ t$. Dashed line: static detuning $\Delta_A/\gamma_c = 0.5$. Dot-dashed line: static detuning $\Delta_B/\gamma_c = 0.25$. Solid and long-dashed lines: periodic sudden shifts of the detuning between $\Delta_A$ and $\Delta_B$. Solid line: starting with detuning $\Delta_A$ (solid arrows in inset). Long-dashed line: starting with detuning $\Delta_B$ (dashed arrows in inset). Frequencies $\omega_A$ and $\omega_B$ are in the vicinity of a discrete mode $\omega_d$ and of a PBG edge $\omega_U$ (inset)

Let us now consider the initial superposition

$$|\Psi(0)\rangle = \alpha(0)\ |e,\{0_\omega\}\rangle + \beta_d(0)\ |g,1_{\omega_d}\rangle \qquad (16)$$

and a non-negligible coupling constant $\kappa_d$. In this case, the periodic dynamic population of the excited state also strongly exceeds the static one. On the other hand, the discrete mode amplitude $\beta_d(t)$ diminishes as compared with the static case. Most importantly, the *instantaneous dynamic fidelity* $|\langle\Psi(0)|\Psi(t)\rangle|^2$ is periodically enhanced as compared to the static one, as demonstrated numerically (dot-dashed line) in Fig. 2 (see Sec. V for quantitative assumptions).

## IV. APPLICATION TO CONTROL-PHASE GATES

In order to use these results for quantum logic gates, let us consider the example of control-phase gate, which consists in shifting the phase of the target-qubit excited state by $\pi$ via interaction with the control qubit [22]. Such a gate is characterized by the truth table [30]

$$
\begin{aligned}
&|0\rangle\ |0\rangle \rightarrow |0\rangle\ |0\rangle,\\
&|0\rangle\ |1\rangle \rightarrow |0\rangle\ |1\rangle,\\
&|1\rangle\ |0\rangle \rightarrow |1\rangle\ |0\rangle,\\
&|1\rangle\ |1\rangle \rightarrow e^{i\pi}\ |1\rangle\ |1\rangle.
\end{aligned}
\tag{17}
$$

In order to ensure compatibility with the proposed decoherence control, the phase shift must be accumulated gradually, so as to preserve the coherence of the system. We have found that a single sudden shift of $\pi$ is incompatible with our method. By contrast, ten or twenty sudden shifts of $\pi/10$ or $\pi/20$, respectively, alternating with an appropriate sequence of detuning changes, keep the fidelity high, with little decoherence. Without attempting to fully optimize the process, we have been able to find such dynamics of the shift that preserves a high fidelity of the system state. The system begins to evolve following the "counter-intuitive" detuning sequence discussed in Sec. III (not to be confused with the adiabatic STIRAP method [24]!). As soon as two sudden changes of the detuning have been performed, the conditional phase shifts of $\pi/10$ or $\pi/20$ take place, to be followed by two more sudden changes and so forth, each time optimizing the detuning to obtain the best protection against spontaneous emission. The total gate operation is completed within the time-interval of maximum fidelity as seen in Fig. 2.

Figure 2, showing the fidelity of the system relative to its initial state during the realization of a control phase gate according to the procedure described above, is perhaps our most impressive finding. We can see that the fidelity is increased using the "counterintuitive" sequence of detunings (solid line) as compared to the static (fixed) choice of maximal detuning (long-dashed line), or compared to the dynamically enhanced fidelity $|\langle\Psi(0)|\Psi(t)\rangle|^2$ obtained without gate operations (dot-dashed line).

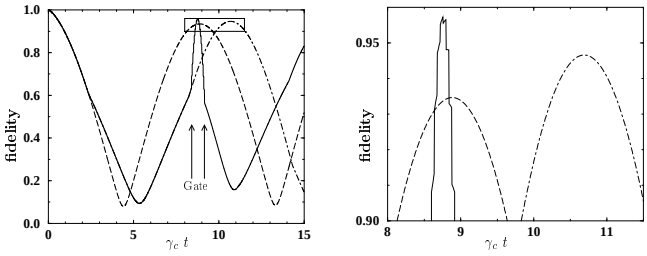


FIG. 2: Fidelity of a superposition state (16) as a function of the dimensionless time $\gamma_c t$. Long-dashed line: static detuning $\Delta_A/\gamma_c = 0.5$. Dot-dashed line: periodic sudden shifts of the detuning from $\Delta_A/\gamma_c = 0.5$ to $\Delta_B/\gamma_c = 0.25$. Solid line: periodic detuning shifts alternating with control phase shifts, effecting the control phase gate. Arrows mark the start and the end of the gate operation. Right panel: enlarged view of the rectangle.

## V. APPROXIMATIONS AND NUMERICAL SOLUTIONS

While the results in Fig. 1 and 2 are generic, they have been obtained based on certain quantitative assumptions. In what follows we outline these assumptions and their numerical implementation.

### A. The effective mass approximation and continuum discretization

In our numerical studies, we have applied the results of the foregoing general analysis to a model density-of-modes (DOM) distribution. This distribution is derived on keeping the lowest term in the Taylor expansion of the dispersion relation $\omega(\vec{k})$ near a cut-off frequency $\omega_U$ in a 3D-periodic structure (photonic, electronic or vibronic crystal). This yields

$$
\omega \approx \omega_U + \sum_{i=x,\ y,\ z} A_i\ (k - k_U)_i^2, \tag{18}
$$

which is known as the effective-mass approximation [32]. In a structure with period $L$, $k_U$ satisfies the Bragg condition $k_U = \pi/L$. The corresponding DOM in a three-dimensionally periodic structure with an allowed point-group symmetry may be approximated as [4]

$$
\rho(\omega) \propto (\omega - \omega_U)^{(1-D)/2}\ \theta(\omega - \omega_U), \tag{19}
$$

where $\theta$ is the step function and $D$ is the dimension of the Brillouin-zone surface spanned by band-edge modes with vanishing group velocity. All parameters and variables in Figs. 1 and 2 are scaled to the effective coupling $\gamma_c = \kappa^2/\sqrt{\epsilon}$, which depends on the edge steepness $\epsilon$.

Results discussed in Secs. III and IV have been obtained in the isotropic dispersion approximation, corresponding to $D = 2$ [5, 6]. In the domain of photonic crystals such an approximation yields qualitatively correct results [6, 17].

The modification of these results to allow for the anisotropy of the density of modes in a realistic photonic crystal can be undertaken using the *anisotropic dispersion relation* with $D = 0$. This leads to a DOM [6, 17]

$$
\rho(\omega) \sim (\omega - \omega_U)^{1/2}. \tag{20}
$$

Figure 3 shows that our results concerning the efficiency of the periodic counterintuitive dynamics remains true

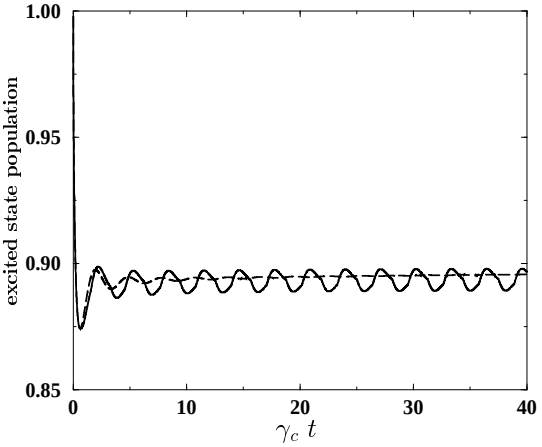


FIG. 3: Excited state population as a function of dimensionless time $\gamma_c\ t$ in the case of an anisotropic density of modes. Initial and final detunings are the same as in Fig. 1: $\Delta_A/\gamma_c = 0.5$ and $\Delta_B/\gamma_c = 0.25$. Long-dashed line: static case for detuning $\Delta_A$. Solid line: periodic counterintuitive dynamics.

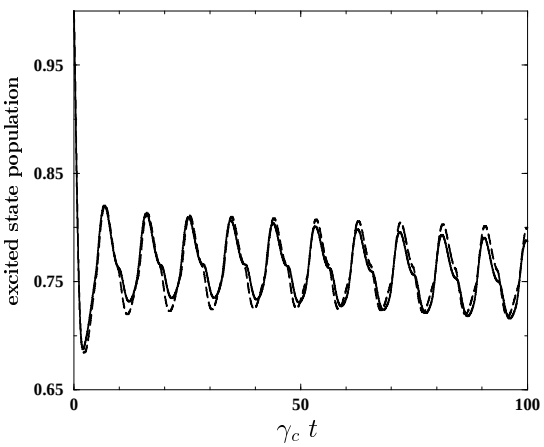


FIG. 4: Influence of finite transition times as compared with the sudden change approximation. Long-dashed line: periodic counterintuitive dynamics in the sudden change approximation (same curve as solid line of Fig. 1). Solid line: same periodic counterintuitive dynamics with finite transition times. Initial and final detunings are $\Delta_A/\gamma_c = 0.5$ and $\Delta_B/\gamma_c = 0.25$.

when considering the anisotropy of the periodic structure. In order to numerically solve the system (5), the continuum has been discretized. The coupling $\kappa(\omega)$ has been approximated to be constant (independent of $\omega$) and real, and the analytic expression (19) or (20) for the DOM has been used.

### B. Validity of the dynamical analysis

Since the sudden change approximation is not realizable experimentally, we have considered the effects of finite transition times between $\Delta_A$ and $\Delta_B$, by using a sequence of pulses that vary as $\exp[-(t-\tau_n)^8/\tau^8]$, *i.e.*, are centered on $\tau_n$ and have half-width $\tau$ of the order of $\tau_A/4 \sim \tau_B/4$. The excited state population is only slightly modified by such finite rise- and falloff-times (Fig. V B). The rotating-wave approximation is obeyed, since the switching time is long enough: $\tau \gg 1/\omega_0$.
We have compared our results, which allow for possibly strong coupling of $|e\rangle$ with the continuum edge, with those of the universal formula of Ref. [28]. This formula expresses the decay rate of $\alpha(t)$ by the convolution of the periodic modulation spectrum and the continuum coupling spectrum. We find good agreement with this formula only in the regime of *weak coupling* to the continuum edge, when the dimensionless detuning parameter $\Delta_{at}/\gamma_c > 5$, as expected from the limitations of the theory in Ref. [28]. We note that the phase-modulation models of Refs. [27] or [29] are not applicable to the present situation.

## VI. DISCUSSION

The following experimental scenarios may be envisioned for demonstrating the proposed control of decoherence. (a) Pairs of qubits are realizable by two species of active rare-earth dopants [31] or quantum dots in a photonic crystal. (b) Alternatively these may be atoms within a high-$Q$ cavity that may have different, *controllable* coupling strengths to the cavity mode. (c) Another candidate system is that of neighboring atoms or ions trapped in optical lattices or ion-arrays when their local vibrational resonances are close to the Debye cutoff of the structure.

It is possible to control the rate of spontaneous emission and the frequency of excited-state oscillations by varying only the two detuning parameters $\Delta_A$ and $\Delta_B$. Let us assume that the transition frequency of one qubit is initially detuned by $\Delta_A \sim 4$ MHz from the continuum edge with coupling constant $\gamma_c \sim 10$ MHz, and by $\sim 3$ MHz from the resonance of the other qubit. This $\Delta_A$ is abruptly modulated by fsec non-resonant laser pulses, which exert $\sim 3$ MHz AC Stark shifts. Between successive shifts, the neighboring qubits are near-resonant with each other and therefore can become entangled (*e.g.*, dipole-dipole coupled), thus effecting the high-fidelity phase-control gate operation [22], as in Fig. 2. The required pulse rate is $\gamma_c/10 \sim 1$ MHz, much lower than the pulse rate stipulated under similar conditions by the previously proposed ultrafast-modulation / quantum-Zeno strategies [27, 28, 29].

To summarize, we have discovered that a "counter-

intuitively" ordered sequence of abrupt changes of the detuning between the qubit transition and a continuum edge (the photonic band cut-off), is able to protect the qubit state from spontaneous emission of photons or phonons more effectively than the intuitively obvious alternative, which is to fix the largest possible detuning value. This method is effective even under conditions of strong coupling to the continuum , as opposed to previously proposed phase- or frequency-modulation strategies [16, 27, 28, 29]. The present method is a highly advantageous means of maintaining high fidelity of quantum states and quantum-logic operations in the presence of decoherence by nonadiabatic interference, contrary to prevailing adiabatic approaches to quantum-state control. This may pave the way to new methods of controlling decay and decoherence in spectrally structured continua.

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